❺ Exerciții rezolvate

Soluție (Exercițiul 2.4)

$\boxed{*}$ Acum, se aplică $\mathcal{F}$ ecuației de mai sus:

$$\Phi_y(\omega) = \sum_{k\in\mathbb{Z}} \sum_{\ell\geq 0} \sum_{m\geq 0} h_\ell h_m r_u[k+m-\ell] e^{-j\omega k} \times e^{-j\omega(m-\ell)} \times e^{+j\omega(m-\ell)} +$$

$$+ \sum_{k\in\mathbb{Z}} \sum_{\ell\geq 0} \sum_{m\geq 0} h_\ell g_m r_{ue}[k+m-\ell] e^{-j\omega k} e^{-j\omega(m-\ell)} e^{+j\omega(m-\ell)} +$$

$$+ \sum_{k\in\mathbb{Z}} \sum_{\ell\geq 0} \sum_{m\geq 0} h_\ell g_m r_{eu}[k+\ell-m] e^{-j\omega k} e^{-j\omega(\ell-m)} e^{+j\omega(\ell-m)} +$$

$$+ \lambda^2 \sum_{k\in\mathbb{Z}} \sum_{\ell\geq 0} \sum_{m\geq 0} g_\ell g_m \delta_0[k+m-\ell] e^{-j\omega k} e^{-j\omega(m-\ell)} e^{+j\omega(\ell-m)} =$$

$$= |H(e^{j\omega})|^2 \Phi_u(\omega) + H(e^{j\omega}) \overline{G(e^{j\omega})} \Phi_{ue}(\omega) +$$

$$+ \overline{H(e^{j\omega})} G(e^{j\omega}) \Phi_{eu}(\omega) + |G(e^{j\omega})|^2 \lambda^2$$

Soluție (Exercițiul 2.4)

Obs

$$\Phi_{ue}(\omega) \triangleq \sum_{k\in\mathbb{Z}} r_{ue}[k]e^{-j\omega k} = \sum_{k\in\mathbb{Z}} r_{eu}[-k]e^{j\omega k} =$$

simetrie încrucișată

$$= \Phi_{eu}(-\omega)$$

$$\Phi_y(\omega) = |H(e^{j\omega})|^2\Phi_u(\omega) + \lambda^2|G(e^{j\omega})|^2 +$$

$$+ H(e^{j\omega})\overline{G(e^{j\omega})}\Phi_{ue}(\omega) +$$

$$+ \overline{H(e^{j\omega})}G(e^{j\omega})\Phi_{ue}(-\omega) \qquad \forall\omega\in\mathbb{R}$$

Termeni principali

Termeni de interferență (paraziți)

❺ Exerciții rezolvate

Soluție (Exercițiul 2.4)

► Evident :

$$\begin{cases} |H(e^{j\omega})|^2 = \dfrac{b}{(1+a^2)+2a\cos\omega} \\ |G(e^{j\omega})|^2 = \dfrac{1}{(1+a^2)+2a\cos\omega} \\ H(e^{j\omega})\overline{G(e^{j\omega})} = \dfrac{b\,e^{-j\omega}}{(1+a^2)+2a\cos\omega} \\ \overline{H(e^{j\omega})}\,G(e^{j\omega}) = \dfrac{b\,e^{+j\omega}}{(1+a^2)+2a\cos\omega} \end{cases}$$

$$\Downarrow$$

$$\Phi_y(\omega) = \frac{1}{(1+a^2)+2a\cos\omega} \times$$

$$\times\left[b^2\Phi_u(\omega) + \lambda^2 + b\left(\Phi_{ue}(\omega)e^{-j\omega} + \Phi_{ue}(-\omega)e^{+j\omega}\right)\right] \quad \forall\,\omega\in\mathbb{R}.$$

● OE [1,1] Similar, căci $G(q^{-1}) \equiv 1$.

$$\Phi_y(\omega) = |H(e^{j\omega})|^2\Phi_u(\omega) + \lambda^2 +$$

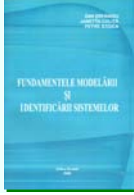
$$+ H(e^{j\omega})\Phi_{ue}(\omega) + \overline{H(e^{j\omega})}\Phi_{ue}(-\omega), \ \forall\,\omega\in\mathbb{R}.$$

&

$$\Phi_y(\omega) = \frac{1}{(1+a^2)+2a\cos\omega}\left[b^2\Phi_u(\omega)+\lambda^2\right] +$$

$$+b\left[\frac{\Phi_{ue}(\omega)}{1+ae^{-j\omega}} + \frac{\Phi_{ue}(-\omega)}{1+ae^{+j\omega}}\right], \ \forall\,\omega\in\mathbb{R}$$

❺ Exerciții rezolvate



Soluție (Exercițiul 2.4)

Răspunsuri în frecvență ideale:

$\boxed{\begin{array}{l}\text{ARX}[1,1]\\ \text{OE}[1,1]\end{array}}$: $$H(e^{j\omega}) = \frac{b e^{-j\omega}}{1 + a e^{-j\omega}}$$

$\boxed{\begin{array}{l}\text{ARX}[2,2]\\ \text{OE}[2,2]\end{array}}$: $$H(e^{j\omega}) = \frac{e^{-j\omega}(b_1 + b_2 e^{-j\omega})}{1 + a_1 e^{-j\omega} + a_2 e^{-2j\omega}}$$

$\forall \omega \in \mathbb{R}$.

- Obs: Aceste relații se obțin imediat aplicând $\mathcal{F}$ secvenței pondere ideale. De exemplu:

$$\sum_{k\geq 0} h_k e^{-j\omega k} \underset{\substack{\uparrow \\ \boxed{\text{ARX}[1,1]} \\ (\text{vezi } \boxed{\text{Ex } 2.2})}}{=} b \sum_{k\geq 1} (-a)^{k-1} e^{-j\omega k} =$$

$$= b e^{-j\omega} \sum_{k\geq 0} (-a)^k e^{-j\omega k} =$$

$$= \frac{b e^{-j\omega}}{1 + a e^{-j\omega}}$$

- Rezultă că răspunsul în frecvență se obține direct înlocuind q^{-1} cu $e^{-j\omega}$ în expresia lui $H(q^{-1})$.