SUBIECTE

Nr 2 :

(1) Cons. sursa binară fără memorie, cu simbolurile echiprobabile. Avem matricea de zgomot.

a) $P(Y/X) = \begin{bmatrix} 5/8 & 1/4 & ? \\ ? & 1/4 & 5/8 \end{bmatrix}$.

$\sum_i p(x_i, y_j) = 1 \Rightarrow \frac{5}{8} + \frac{1}{4} + ? = 1 \Rightarrow g_1 = 1/8 = g_2. \Rightarrow$

$\Rightarrow P(Y/X) = \begin{bmatrix} 5/8 & 1/4 & 1/8 \\ 1/8 & 1/4 & 5/8 \end{bmatrix}$

Canal uniform după intrare : ∃ permutări pe linie.

b) Calculați $H(Y)$, $H(Y/X)$, $H(X,Y)$, $H(X/Y)$, $I(X,Y)$, C.

$\boxed{P(X,Y) = P(Y/X) \cdot P(X)}$

$P(X,Y) = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 5/8 & 1/4 & 1/8 \\ 1/8 & 1/4 & 5/8 \end{bmatrix} = \begin{bmatrix} 5/16 & 1/8 & 1/16 \\ 1/16 & 1/8 & 5/16 \end{bmatrix}$.

$p(x_1) = \sum_{j=1} p(x_i, y_j) =$ $\qquad p(y_1) = \sum_{i=1} p(x_i, y_j)$.

$p(x_2) = \sum_{j=1} p(x_i, y_j) =$. $\qquad p(y_2) = ---$

$p(y_3) = ---$

$H(Y) = -\sum_j p(x_i, y_j) \cdot \log p(x_i, y_j)$

$H(X,Y) = -\sum_i p(x_i, y_i) \log p(x_i, y_i)$.

$H(X/Y) = H(X,Y) - H(Y)$, $I(X,Y) = H(X) + H(Y) - H(X,Y)$.

$C = \log m + \sum_{i=1}^{m} p(x_i, y_j) \log p(x_i, y_j)$

c) $l = ?$ $P_e = -10 dB$, $[S/Z]_{dB} = 20$, $\alpha = 3\ dB/km$, $Z_0 = -40 dB$.

$$\boxed{10 \log \frac{P_{out}}{P_e} = -\alpha \cdot l} \Rightarrow 10 \log P_{out} - 10 \log P_e = -\alpha l \Rightarrow$$

$$\Rightarrow l = \frac{10 \log P_{out} - 10 \log P_e}{-\alpha} = \frac{[P_e]_{dBm} - [P_{out}]_{dBm}}{\alpha}$$

$$[S/Z]_{dB} = 10 \log \frac{S}{Z} \Rightarrow 10 \log S - 10 \log Z = [S/Z]_{dB}.$$

$$\Rightarrow [S]_{dBm} - [Z]_{dBm} = [S/Z]_{dB}$$

$$l = \frac{[P_e]_{dBm} - \left[\frac{S}{Z}\right]_{dB} - [Z_0]_{dBm}}{\alpha} = \frac{-10 - 20 + 40}{3} = \frac{10}{3}\ km.$$

2. Semnal modulat amplitudine:

$$S_{MA}(t) = (7 + \sin 400\pi t - 20 \cos 800\pi t) \cos 10^4 \pi t$$

a) Expresiile temporale ale purtătoarei și semnalului modulator.

$S_p = ?$, $S_M = ?$

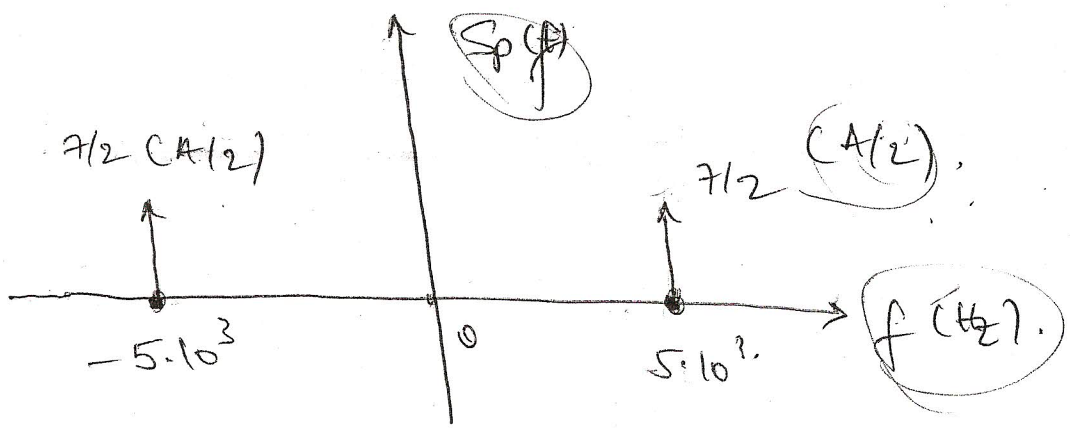
$$\boxed{S_{MA}(t) = (A_p + s_m(t)) \cos 2\pi f_p t} \Rightarrow$$

$$\Rightarrow \begin{cases} A_p = 7 \\ f_p = \frac{10^4}{2} = 5 \cdot 10^3\ Hz \end{cases}$$

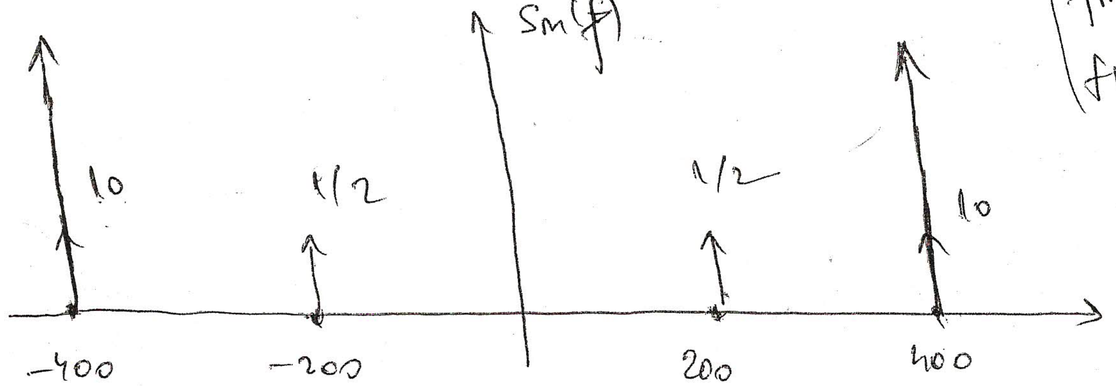
$$\boxed{S_p(t) = A_p \cos(2\pi f_p t)} \Rightarrow S_p(t) = 7 \cos 10^4 \pi t$$

$$S_M(t) = \sin 400\pi t - 20 \cos 800\pi t.$$

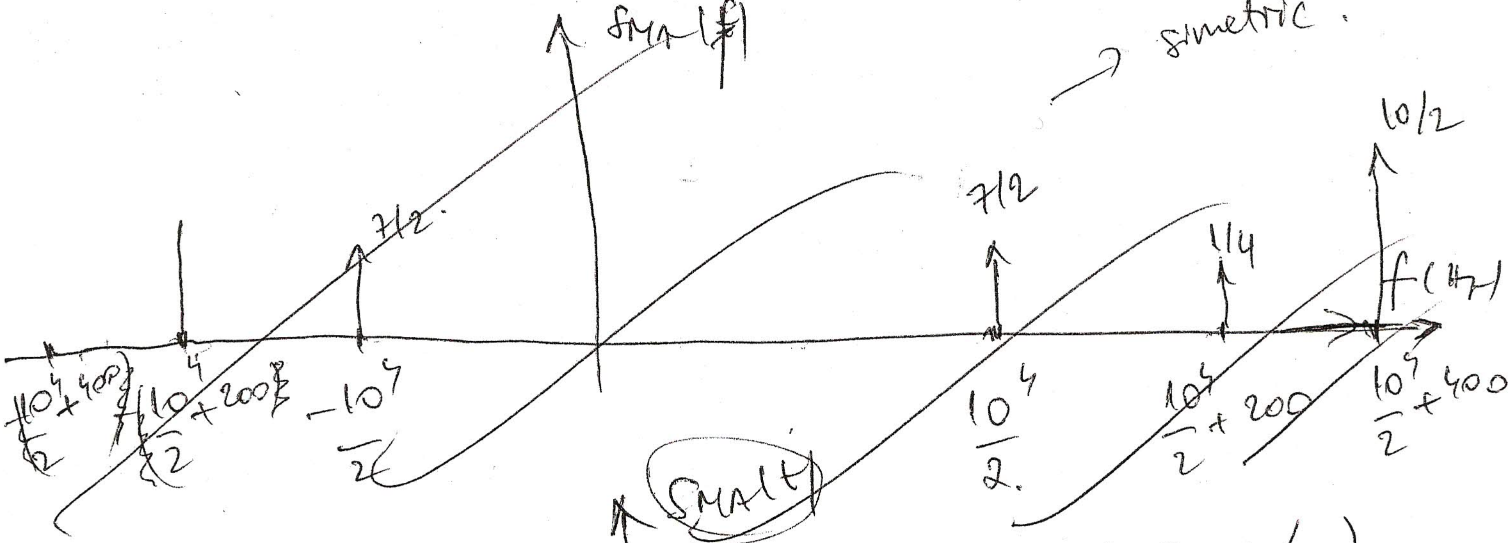
b) Spectrele semnalelor: S_m, S_p, S_{MA}.



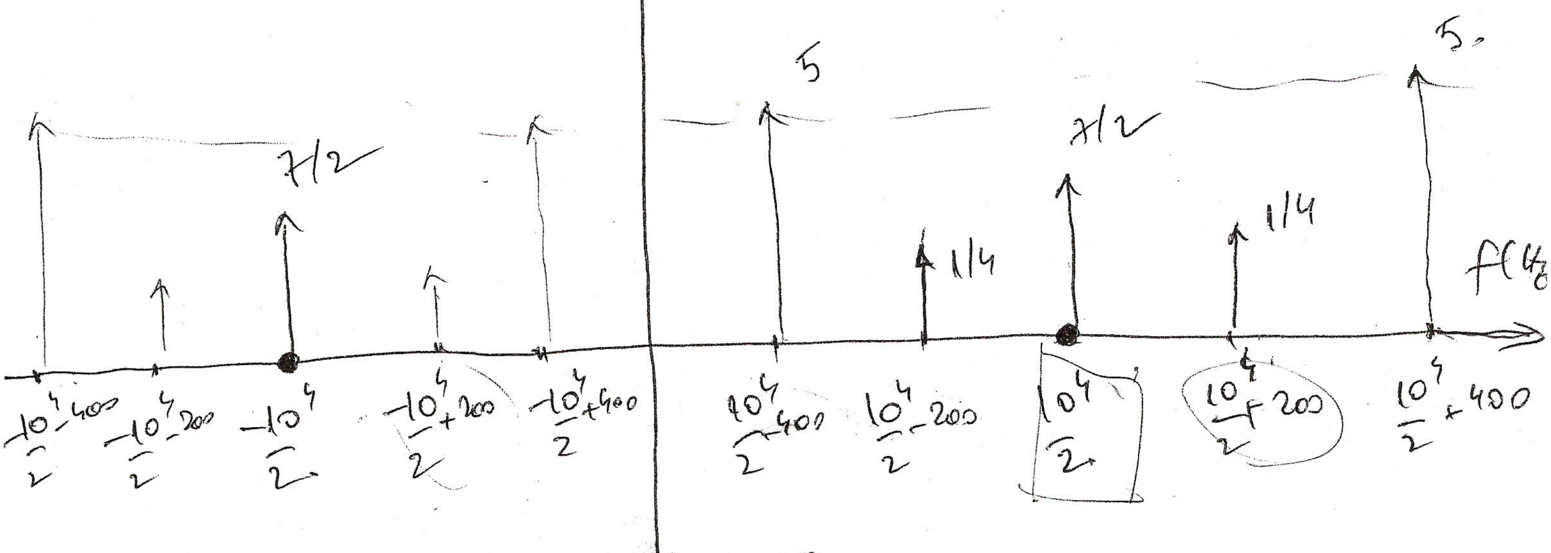
$f_{m_1} = 200$
$f_{m_2} = 400$.



→ simetric!



(se ia amplit. Sm(f)/2)
pt. pete 200, 400.



c) Calculați indicele modulației și eficiența transmisiei.

$m = ?$, $\eta = ?$

$$\boxed{m = \frac{A(t)_{max} - A(t)_{min}}{A(t)_{max} + A(t)_{min}}}$$

$S_{MA}(t) = A(t)\cos\omega_p t$

$A(t) = 7 + \sin 400\pi t + \cancel{2} \cos 800\pi t$

$A'(t) = 400\pi\cos 400\pi t - 1600\pi \sin 800\pi t = 0 \Rightarrow$

$\Rightarrow \cos 400\pi t = 4 \sin 800\pi t \Rightarrow \cos 400\pi t = 4 \cdot 2 \sin 400\pi t \cdot \cos 400\pi t$ $\neq 0$.

$\Rightarrow \frac{1}{8} = \sin 400\pi t$. $\frac{1}{8}$

$\cos 400\pi t \neq 0$. ± 1. (cond $\cos \neq 0$).

$\cos 2x = 1 - 2\sin^2 x$

$A(t)\Big|_{\sin 400\pi t = 1} = 7 + 1 + 2 \cdot (-1) = 6$.

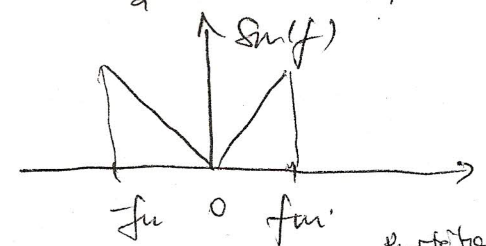
$A(t)\Big|_{\sin 400\pi t = -1} = 7 - 1 - 2 = 4$

$A(t)\Big|_{\sin 400\pi t = 1/8} = 7 + \frac{1}{8} + 2 \cdot \frac{31}{32} =$ ~~$7 + 0.125 + 19.93 \approx 28$~~. $= 7 + 0.125 + 1.93 \approx 9.05$.

$A(t)_{min} = 4$, $A(t)_{max} = 9$

$m = \frac{4/1}{13} = \cancel{3,15} \approx \cancel{31,5\%} = \frac{9-4}{9+4} = \frac{5}{13} \approx 0.38 \Rightarrow m = 38\%$.

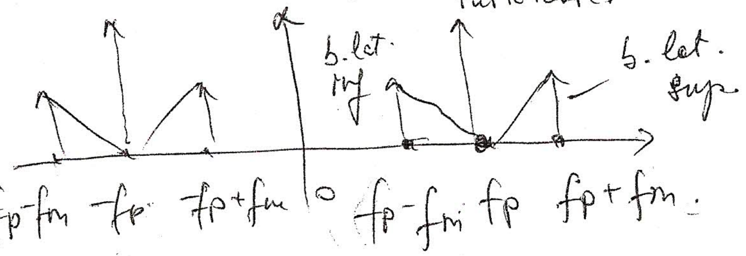
$$\boxed{\eta = \frac{m^2}{m^2+2}}$$ (avem 2 benzi laterale).



1 bandă!

$$\boxed{\eta = \frac{m^2}{2(m^2+2)}}$$

$\eta = \frac{0.14}{4,14} = 0.03 = 3\%$



d) Propuneți o soluție de demodulare coerentă cu precizarea caracteristicilor blocurilor fol.

③ Cons. următorul text:

ACCES_NELIMITAT_LA_ACESTE_TESTE_SI_LA_ALTELE_CA_ELE.

A: 7/51 N: 1/51 T: 6/51

C: 4/51 L: 6/51 Spațiu: 9/51.

E: 10/51 I: 3/51

S: 4/51 M: 1/51

a) probabilitățile de apariție:

p(A) = 0.13 p(N) = 0.019

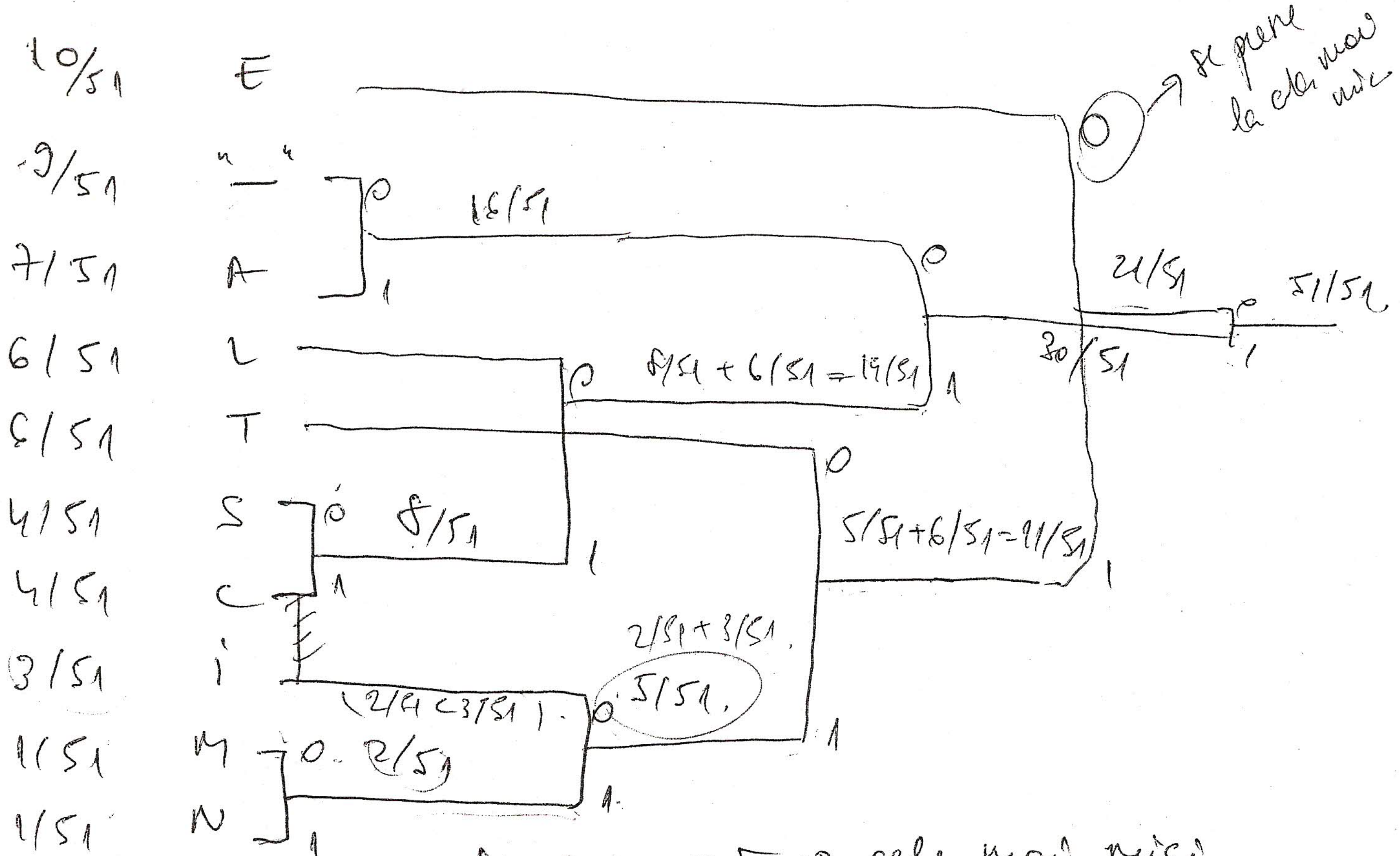
p(C) = 0.07 p(sp) = 0.17

p(E) = 0.19 p(M) =

p(S) = 0.07

b) Codificați cu Huffman:

Luăm probabilit. și ordonăm descrescător!



se grupează 2 cele mai mici.

Tabel cod.

E	00
_	100
A	101
L	110
T	010
S	1110
C	1111
I	0110
M	01110
N	01111

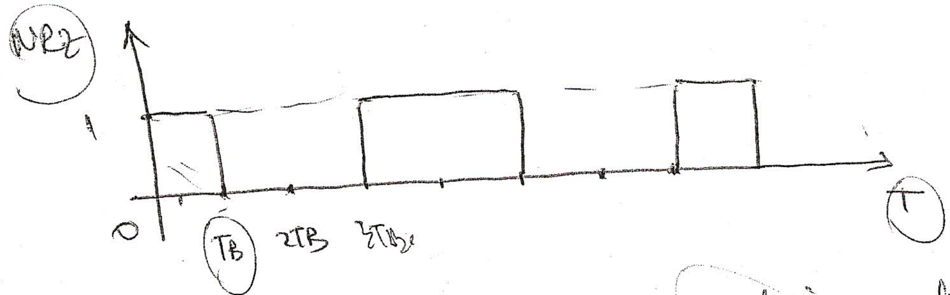
(2) Impuls neegalizat transmis în sistem transm. a semnalelor binare în bandă de bază b-fazic valorile la interv. eșantionare.

K	-3	-2	-1	0	1	2	3
Pr(K)	0	0	0.1	0.9	0.1	0	0

a) semnalul emis în linie, la intrare în codor se aplică

10011001.

NRZ



b) proiectați egalizator cu 3 etaje ce forțează răspuns în 0 la $K = \pm 1, \pm 2, \pm 3$ și în 1 la $K=0$. $\boxed{2N+1=3} \Rightarrow \boxed{N=1}$

0 0 0 1 0 0 0

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} pr(0) & pr(-1) & pr(-2) \\ pr(1) & pr(0) & pr(-1) \\ pr(2) & pr(1) & pr(0) \end{bmatrix} \cdot \begin{bmatrix} C_{-1} \\ C_0 \\ C_1 \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0.1 & 0.9 & 0.1 \\ 0 & 0.1 & 0.9 \end{bmatrix} \cdot \begin{bmatrix} C_{-1} \\ C_0 \\ C_1 \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 9 & 1 & 0 \\ 1 & 9 & 1 \\ 0 & 1 & 9 \end{bmatrix} \begin{bmatrix} C_{-1} \\ C_0 \\ C_1 \end{bmatrix}.$$

inversăm matricea.

$9 \quad A^{-1} \quad 0$

$9A^T = 9A$.

$$A^{-1} = \frac{A^*}{\Delta}$$

$$A^{*} = \begin{bmatrix} 80 & -9 & 1 \\ 9 & 81 & -1 \\ 1 & -9 & 80 \end{bmatrix} \Rightarrow A^{-1} = \frac{A^{*}}{711} \cdot \frac{1}{10}.$$

$$A^{-1}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \frac{1}{10}\begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} \Rightarrow 10A^{-1}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} \Rightarrow$$

$$\Rightarrow \begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \frac{10}{711}\begin{bmatrix} -9 \\ 8 \\ -9 \end{bmatrix} = \begin{bmatrix} -0.12 \\ 1.13 \\ -0.12 \end{bmatrix}$$

$$peg(k) = \sum_{n=-N}^{N} c_n \cdot pr(k-n). \Rightarrow$$

$$\Rightarrow peg(k) = \sum_{n=-1}^{1} c_n\, pr(k-n).$$

$\underline{k=0}$ $\quad peg(0) = c_{-1}\,pr(1) + c_0\,pr(0) + c_1\,pr(-1) =$

$\quad -0.12 \cdot 0.1 + 1.13 \cdot 0.9 - 0.12 \cdot 0.1$

$\quad = -0.024 + 1.017 = 0.993 \approx 1.$

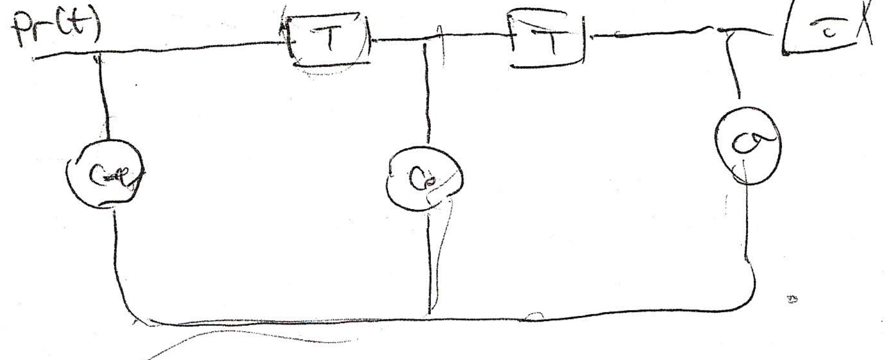
$\underline{k=\pm 1}$ $\quad peg(1) = c_{-1}\,pr(2) + c_0\,pr(1) + c_1\,pr(0) =$

$\quad -0.12.$

$\quad = 0.$

$\underline{k=\pm 2}$ $\quad peg(2) = 0, \quad \underline{k=\pm 3}$ $\quad peg(3) = 0.$

c) Calculati $p_e(k)$.



⑥. Un canal are matricea de zgomot:

$$P(X/Y) = \begin{bmatrix} 1/2 & 1/3 & q_1 \\ q_2 & 2/3 & 1/4 \end{bmatrix}.$$

$P(X,Y)$

$H(X/Y)$

a) $\frac{1}{2} + \frac{1}{3} + q_1 = 1 \Rightarrow 5 + 6q_1 = 6 \Rightarrow q_1 = \frac{1}{6}.$

$q_2 + \frac{2}{3} + \frac{1}{4} = 1. \Rightarrow 11 + 12 q_2 = 12 \Rightarrow q_2 = \frac{1}{12}.$

$$\Rightarrow P(X/Y) = \begin{bmatrix} 1/2 & 1/3 & 1/6 \\ 1/12 & 2/3 & 1/4 \end{bmatrix}.$$

b) $H(X,Y)$ pt. simboluri echiprobabile la intrare

$P(X/Y) \,|: p(y) \Rightarrow \boxed{P(X,Y)} =$

$$\Rightarrow P(X,Y) = \begin{bmatrix} \frac{1}{2}\cdot\frac{1}{2} & \frac{1}{2}\cdot\frac{1}{3} & \frac{1}{2}\cdot\frac{1}{6} \\ \frac{1}{2}\cdot\frac{1}{12} & \frac{1}{2}\cdot\frac{2}{3} & \frac{1}{2}\cdot\frac{1}{4} \end{bmatrix} = \begin{bmatrix} 1/4 & 1/6 & 1/12 \\ 1/24 & 1/3 & 1/8 \end{bmatrix}.$$

$P(x,y) = P(X/Y) \cdot P(X).$

$p(x) = 1/2$. pt. ambele evenimente

$$\Rightarrow H(X,Y) = -\left[\frac{1}{4}\cdot\log_2\frac{1}{4} + \frac{1}{6}\cdot\log_2\frac{1}{6} + \frac{1}{12}\cdot\log_2\frac{1}{12} + \frac{1}{24}\cdot\log_2\frac{1}{24} + \frac{1}{3}\cdot\log_2\frac{1}{3} + \frac{1}{8}\cdot\log_2\frac{1}{8}\right].$$

⑦. Pt. un canal discret:

$$P(X,Y) = \begin{bmatrix} 0.1 & 0.25 \\ 0.2 & 0 \\ 0.3 & 0.15 \end{bmatrix}.$$

a) $H(X)$, $H(Y)$, $H(X/Y)$, $H(Y,X)$.

b) $H(X,Y)$, c) $I(X,Y)$ transinformația.

$P(Y,X)$ $P(X)$ $P(X,Y)$

Obs! Proprietăți $P(X,Y)$.

- suma elementelor pe linie:

$$\sum_{j=1}^{M} p(x_i, y_j) = p(x_i) \quad cu \quad \sum_{i=1}^{n} p(x_i) = 1.$$

deci $p(x_1) = 0.35$, $p(x_2) = 0.2$, $p(x_3) = 0.45$.

$$H(X) = -[p(x_1)\cdot \log_2 p(x_1) + p(x_2)\cdot \log_2 p(x_2) + p(x_3)\cdot \log_2 p(x_3)]$$

$$= -[0.35 \log_2 0.35 + 0.2 \cdot \log_2 0.2 + 0.45 \cdot \log_2 0.45].$$

$$H(Y) = -[p(y_1)\cdot \log_2 p(y_1) + p(y_2)\cdot \log_2 p(y_2)].$$

- suma elementelor pe coloană:

$$\sum_{i=1}^{n} p(x_i, y_j) = p(y_j) \cdot \quad cu \quad \sum_{j=1}^{m} p(y_j) = 1.$$

$p(y_1) = 0.6$ și $p(y_2) = 0.4$.

$H(X,Y)$ se obține din $P(X,Y)$.

Relațiile!
$H(Y/X) = H(X,Y) - H(X)$. $\Rightarrow$ $H(Y/X) = \dots$ biți
$H(X/Y) = H(X,Y) - H(Y)$. $\Rightarrow$ $H(X/Y) = \dots$ biți.

$I(X,Y) = H(X) + H(Y) - H(X,Y) = \dots$ biți

(8) Pt canal gaussian cu $B = 1MHz$ și $S/Z = 30dB$.

Formulă! $\left[\frac{S}{Z}\right]_{dB} = 10 \log_{10} \frac{S}{Z} \Rightarrow \frac{S}{Z} = 10^{\frac{S}{Z}\cdot\frac{1}{10}}$

$\left[\frac{S}{Z}\right]_{dB} = 30\ dB. \Rightarrow \boxed{S/Z = 10^3}$

Calculați capacitatea și timpul în care se transmite pe canal 1 milion de caractere ASCII (8 biți)?

$$C = B \log_2\left(1+\frac{S}{Z}\right) = 1 \cdot \log\left(1+10^3\right) = 9.97 \text{ Mbps}$$

$10^6 \cdot 8$ biți.

$I = \frac{C}{\overline{\tau}} \cdot \tau \Rightarrow \quad \tau = \frac{I}{C} = \frac{10^{6\,3} \cdot 8}{9{,}97 \cdot 10^3} = 802{,}5\ s$

$\overline{\tau}$ - timpul.

⑨. $B = 4$ KHz

$f_E = 2{,}5 f_N$ (f_N: frecv. Nyquist) frecv. de eșantionare

- fiecare eșantion cuantizat pe 256 niveluri
- eșantioane indep.

a) viteza de transmitere a info.

b) eroarea de ~~transmisie~~ respre sursei zero? pe canal gaussian cu banda de 50 KHz și rap. $S/Z = 23$ dB. ?

c) B necesar transmisie fără eroare dacă $S/Z = 10$ dB ?

a) Criteriu Nyquist.

Pe canal cu B limitat se pot transmite cel mult 2 impulsuri (simboluri)/s pt. fiecare hertz de lungime de B.

$f_E = 2.5 f_N \Rightarrow$ 2 simboluri/s ... 2,5 Hz

$4 \cdot 10^3$ Hz (B).

x simb/s ...

$x = \frac{2 \cdot 4 \cdot 10^3}{2.5} = \frac{8000}{2.5} = 3200$ bit/s.

(V_S).

b) $C =$

Pork

Pl

10. $\ell = \dfrac{10\log_{10}\left(\frac{P_{out}}{P_{in}}\right)}{-A} \Rightarrow 10\log_{10}\left(\frac{P_{out}}{P_{in}}\right) = -A\cdot\ell$

$\dfrac{-[P_{out}]_{dBm} + [P_{in}]_{dBm}}{A} = \ell$ (S, N)

$[N_{et}]_{dBm} = -40\,dBm$

$[P_{in}]_{dBm} = -10\,dBm$

$(S/N) = 10\,dB = [S]_{dBm} - [N]_{dBm}$

$$\boxed{\ell = \frac{\left(\frac{S}{N}\right)_{dB} + [N]_{dBm} - [P_{in}]_{dBm}}{-A}} \Rightarrow$$

$\Rightarrow \ell = -\frac{1}{3}\cdot(10 - 40 + 10) = \frac{+20}{3} \approx 7\,km.$

Pt. marjă suplimentară: $+6\,dB$.

$\ell' = \frac{1}{-3}(10 + 6 - 40 + 10) = \frac{14}{3} \approx 5\,km.$

marjă $[M]_{dB}$

c) lungimea medie a cuvântului de cod.

$$\boxed{\overline{n}_{med} = \sum_{1}^{10} p_i \cdot n_i}, \quad n_i \text{ - lungimea } i.$$

$$= \frac{10}{51} \cdot 2 + \frac{9}{51} \cdot 3 + \frac{7}{51} \cdot 3 + \frac{6}{51} \cdot 3 + \frac{6 \cdot 3}{51} + \frac{4}{51} \cdot 8 + \frac{3}{51} \cdot 4 +$$

$$+ \frac{2}{51} \cdot 5 = \ldots$$

d) eficiența în rap cu ASCII extins (8 biți).

$$\eta = \frac{\overline{n}_{min}}{\overline{n}} = \frac{H}{\overline{n} \log 2}, \quad H = \sum_{1}^{10} p_i \log p_i.$$

$g = 2$. (0 și 1 repr.)

e) Varianta de cod de corecție în cazul unei singure erori.

Nr 1

3) a) $x^4 + x + 1$ cons. polinom generator pt cod ciclic (15, 11).

15 biți totali $n = 15$, 11 biți efectiv folosiți, 4 - control paritate.

$$\begin{cases} n = 15 \\ k = 4 \\ m = 11 \end{cases}$$

Regulă. $x^4 + x + 1$ divide $x^{15} - 1$ în modulo 2.

$$x^{15}-1 \quad \Big| \quad x^4+x+1$$

$$x^{11}+x^8+x^7+x^5+2x^4+x^3-x^2-3x-3$$

$$0 \qquad 1$$

$$-\cancel{x^{15}}-x^{12}-x^4$$

$$-x^{12}-x^4-1$$

$$x^{12}+x^9+x^8$$

$$\cancel{x^9-x^4+x^8-1}$$

$$\cancel{-x^9+x^6+x^5}$$

$$\cancel{x^8+x^6+x^4-x^5-1}$$

$$-x^4$$

$$-\cancel{x^4}+x^9+x^8-1$$

$$\cancel{x^4}+x^8+x^7$$

$$\cancel{x^9}+2x^8+x^7-1$$

$$-\cancel{x^9}-x^6-x^5$$

$$2x^8+x^7-x^6-x^5-1$$

$$-2\cancel{x^8}-2x^5-2x^4$$

$$x^7-x^6-3x^5-2x^4-1$$

$$-\cancel{x^7}-x^4-x^3$$

$$-\cancel{x^6}-3x^5-3x^4-x^3-1$$

$$\cancel{x^6}+x^3+x^2$$

$$-3\cancel{x^5}-3x^4+x^2-1$$

$$3\cancel{x^5}+3x^2+3x$$

$$-3\cancel{x^4}+4x^2+3x-1$$

$$3\cancel{x^4}+3x+3$$

$$4x^2+6x+2$$

$$0 \quad 0 \quad 0$$

$$x+x$$

$$2x$$

$$2x \to 0$$

$$x+x=0$$

$$x^{15}-1=(x^4+x+1)(x^{11}+x^8+x^7+x^5+x^3+x^2+x+1)$$

divide!

b) Perechea G, H pt. codul cu polinom. gen x^4+x+1.

$g(x) = x^4+x+1 = g_0[x^k]+\dots+g_k$ (CS).

$$G_{(4,15)} = \begin{bmatrix} 0 & 0 & \dots & 0 & g_0 & \dots & g_k \\ 0 & 0 & \dots & g_0 & \dots & g_k & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & g_0 & \dots & g_k & 0 & \dots & 0 \\ g_0 & \dots & g_k & 0 & \dots & \dots & 0 \end{bmatrix}$$

(m×n)

4x15.

$$G_{(11,15)} = \left[\begin{array}{ccccc|c|cccc} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

H determinată de polinomul ct.

$h(x) = x^{11}+x^8+x^7+x^5+\cancel{x^4}+x^3+x^2+x+1 =$

$\boxed{x^{n-1} = g(x)\cdot h(x)}$ $= h_m x^m + \dots + h_0$.

$$H_{k,m} = \left[\begin{array}{cccc|cccc} h_0 & h_1 & \dots & h_m & 0 & \dots & \dots & 0 \\ 0 & h_0 & h_1 & \dots & h_m & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & h_0 & h_1 & \dots & h_m \end{array}\right]$$

(4, 15)

$$H(4,12) = \begin{bmatrix} \; & - & - & \; \end{bmatrix}.$$

c) Combinațiile : $u_0 = \overset{a_1}{1}\,0\,0\,\overset{a_4}{1}\,\overset{a_5}{1}\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0$

$u_1 = 1\,\overset{a_2}{0}\,\overset{a_3}{0}\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1$

$\in x^4 + x + 1 \ldots$

$$H(4,15) = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$u_0 = \begin{bmatrix} a_1 \\ | \\ | \\ | \\ a_{15} \end{bmatrix}$$

$\cancel{u_0 \cdot H(4,12)} = 0$ dacă aparține codului.

$H(4,15)\,u_0 = 0$

~~$a_1 + a_4$~~ + nr. par

$\underline{a_1} + a_2 + a_3 + \underline{a_4} + a_6 + a_8 + a_9 + a_{12} = 0$

$a_2 + a_3 + \underline{a_4} + \underline{a_5} + a_7 + a_9 + a_{10} + a_{13} = 0$

$a_3 + \underline{a_4} + \underline{a_5} + a_6 + a_8 + a_{10} + a_{11} + a_{14} = 0$

nr. par. pt. u_1. (1 fără)

u_0 aparține

u_1 nu aparține.

d) Redundanța codului

$\rho = 1 - \eta$ eficiența

$\eta = \frac{m}{n} = \frac{11}{15}$

$2^{k} \simeq n$

$k = \log n \quad m = \cancel{k}\ n - \log n$

e) Biți control pt. ciclic (15, 11) să asigure $d_H = 5$!

cod impar $\Rightarrow 2r + 1 = 5 \rightarrow$

$\Rightarrow \boxed{r = 2}$ detect. 2 erori corectare $r-1 = 1$ eroare

corector de lungime g : $g = \cancel{1}$ $g \cdot r = grad(g(x))$

$(15, 11)$ $n = 15$, $m = 11$, $k = 4$.

$r = 4$

$2^4 - 1 = n = 15$

$g(x) = x^4 + x + 1$, $grad(g(x)) = 4$

$grad(g(x)) = 4 = g \cdot r$ cifre test.

$2^r - 1$ < Corect. 1 eroare / Detect. 2 erori

$d_H = 5 \rightarrow$ DET : 2 erori
COR : 1 eroare

pachete. $k \leq 4$.

$\boxed{2r + 1 : -det}$

Cod.

$\boxed{r - 1 \ - corect}$

$(2^r - 1,\ 2^r - gr - 1)$

$2g = 2$ cel mult g erori corectate. $g = 1$.

$(15, 10)$

$g \cdot r = 4 \Rightarrow$ altă notație $\boxed{r = 4}$.

$k = n - m = 4 \geq d_H$

$\Rightarrow 5 \geq d_H = 5$?

Cod corector de lungime:

$2^{r} - 1 = 15$ $\rightarrow$ Corect.: 1 er.
$\rightarrow$ Detect: 2 er.
$\rightarrow$ Pachete: $\leq g \cdot r \Rightarrow 4$.

Codul rot: $(2^r-1, 2^r-2r-1) = (15, 11)$

Pachete de lungime $2g=4$. $\Rightarrow$ biti de control. min $2g$.

$\boxed{k = n-m \geq d}$ pachete. lungime. $d=4$ $15-11=4 \geq 4.$

SUB: 22.06.

① Sursă binară fără memorie, simbolurí echiprobabile:

$$P(Y/X) = \begin{bmatrix} 5/8 & 1/4 & ? \\ ? & 1/4 & 5/8 \end{bmatrix} \begin{matrix} =1 \\ =1 \end{matrix} \quad (\log 3 = 1.6,\ \log 5 = 2.3).$$

a) $\sum_j p(x_i, y_j) = 1 \Rightarrow \frac{5}{8} + \frac{1}{4} + q_1 = 1 \Rightarrow 7 + 8q_1 = 8 \Rightarrow q_1 = \frac{1}{8}$

$\sum_i p(x, y_i) = 1 \Rightarrow q_2 + \frac{1}{4} + \frac{5}{8} = 1 \Rightarrow q_2 = \frac{1}{8}$.

$$P(Y/X) = \begin{bmatrix} 5/8 & 1/4 & 1/8 \\ 1/8 & 1/4 & 5/8 \end{bmatrix}$$

log

$p(x_i)$.

– uniform față de intrare

b). calc. $H(Y)$, $H(Y/X)$, $H(X,Y)$, $H(X/Y)$, $I(X,Y)$, C

$$\cancel{H(Y)} = P(Y/X) = \begin{bmatrix} 5/8 & 1/4 & 1/8 \\ 1/8 & 1/4 & 5/8 \end{bmatrix}.$$

$$P(X,Y) = P(X) \cdot P(Y/X) = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 5/8 & 1/4 & 1/8 \\ 1/8 & 1/4 & 5/8 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{5}{16} & \frac{1}{8} & \frac{1}{16} \\ \frac{1}{16} & \frac{1}{8} & \frac{5}{16} \end{bmatrix} =$$

$P(Y/X)$.

$$H(Y) = \sum_1^3 p_{y_i} \log_2 p_{y_i} = -\left[\frac{6}{16} \cdot \log_2 \frac{6}{16} + \frac{2}{8} \quad \_\_\_ \right].$$

$$H(X) = \sum_1^2 p_{x_i} \log_2 p_{x_i} =$$

$$C = \log m + \sum_{j=1}^{m} p(y_j/x_j) \cdot \log$$

m = nr y-ui

$$H(X,Y) = \sum_{i=1}^{2} \sum_{j=1}^{3} p(x_i, y_j)$$

$$C = \log 3 + [p(y_1/x_1) \cdot \log p(y_1/x_1) + p(y_2/x_2) \cdot \log p(y_2/x_2)$$

c)

$$\frac{27}{128} \cdot 2 = \frac{27}{64} \qquad 0$$

$$\frac{27}{64} \cdot 2 = \frac{27}{32} \qquad 0$$

$$\frac{27}{32} \cdot 2 = \frac{27}{16} = 1, \qquad 1$$

$$\frac{11}{16} \cdot 2 = \frac{11}{8} \qquad 1$$

$$\frac{3}{8} \cdot 2 = \frac{3}{4} \qquad 0$$

$$\frac{3}{4} \cdot 2 = \frac{3}{2} \qquad 1$$

$$\frac{1}{2} \cdot 2 = 1 \qquad 1$$

10/51	E	32/51		000
9/51	_			001
7/51	A			010
6/51	L			011
6/51	T	10/51		100
4/51	S			101
4/51	C			110
3/51	I	9/51		1110
1/51	M	29/51		11110
1/51	N			11111

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