Problemă. Să se calculeze un compensator stabilizator pentru sistemul descris de:

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & -1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

Unde $\lambda_d = [-1 \quad -1 \quad -1]$ și $\lambda_e = [-2 \quad -2 \quad -2]$.

Să se calculeze:

a) F astfel încât $\Sigma(A + BF) = \Lambda_d$.................... 4p
b) Estimatorul Kalman alocat pentru λ_e........... 3p
c) Compensatorul stabilizator............................2p

Oficiu...........1p
Total............10p

Rezolvare:

a) Calculez matricea de controlabilitate:

$$R = [B \;\; AB \;\; A^2B] = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -2 & -2 & -2 \\ 1 & 1 & 2 & 2 & 3 & 2 \end{bmatrix}$$

$$F_0 = 0, g = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$A_0 = A + BF_0 = A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$b_0 = Bg = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$R_0 = [b_0 \;\; A_0b_0 \;\; A_0^2b_0] = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & -2 \\ 1 & 2 & 2 \end{bmatrix}, rang(R_0) = 2 => necontrolabil\breve{a}$$

$$F_0 = 0, g = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$A_0 = A + BF_0 = A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$b_0 = Bg = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$R_0 = [b_0 \;\; A_0b_0 \;\; A_0^2b_0] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow R_0^{-1} = R_0$$

$$R_0^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix} \rightarrow q^T = [0 \quad 0 \quad 1]$$

$$f^T = -q^T \chi_d(A_0) = -q^T (A_0 + I)^3 = -[0 \quad 0 \quad 1] \cdot \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & 3 \end{bmatrix}^3$$

$$= -[0 \quad 1 \quad 3] \cdot \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & 3 \end{bmatrix}^2 = -[1 \quad 4 \quad 8] \cdot \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & 3 \end{bmatrix} = -[5 \quad 12 \quad 20]$$

$$F = F_0 + g \cdot f^T = \begin{bmatrix} 1 \\ -1 \end{bmatrix} [-5 \quad -12 \quad -20] = \begin{bmatrix} 5 & 12 & 20 \\ -5 & -12 & -20 \end{bmatrix}$$

b) L astfel încât $\sigma(A + LC) = \Lambda_e$.

Facem algoritmul de alocare pentru $A_1 = A^T$ și $b_1 = C^T$.

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 2 \end{bmatrix}, \ b_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \text{(Facem } \sigma(A_1 + b_1 f_1^T) = \Lambda_e\text{)}$$

$$R_1 = [b_1 \ \ A_1 b_1 \ \ A_1^2 b_1] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix} \to R_1^{-1} = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 1 & 0 \\ \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix} \to q_1^T = [1 \quad 0 \quad 0]$$

$$f_1^T = -q_1^T (A_1 + 2I)^3 = -[1 \quad 0 \quad 0] \cdot \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 4 \end{bmatrix}^3 = -[2 \quad 1 \quad 0] \cdot \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 4 \end{bmatrix}^2$$

$$= -[4 \quad 4 \quad 1] \cdot \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 4 \end{bmatrix} = -[8 \quad 11 \quad 8]$$

$$L = \begin{bmatrix} -8 \\ -11 \\ -8 \end{bmatrix}$$

$$J = A + LC = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix} + \begin{bmatrix} -8 \\ -11 \\ -8 \end{bmatrix} [0 \quad 0 \quad 1] = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 0 & -8 \\ 0 & 0 & -11 \\ 0 & 0 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -8 \\ 1 & 0 & -12 \\ 0 & 1 & -6 \end{bmatrix}$$

$$K = -L = \begin{bmatrix} 8 \\ 11 \\ 8 \end{bmatrix}, \quad H = B = \begin{bmatrix} 0 & -1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}, \quad M = I_3, \quad N = 0$$

c) $A_c = J + H \cdot F \cdot M = \begin{bmatrix} 0 & 0 & -8 \\ 1 & 0 & -12 \\ 0 & 1 & -6 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} -5 & -12 & -20 \\ 5 & 12 & 20 \end{bmatrix} = \begin{bmatrix} -5 & -12 & -28 \\ 1 & 0 & -12 \\ 0 & 1 & -6 \end{bmatrix}$

$$B_c = K + H \cdot F \cdot N = \begin{bmatrix} 8 \\ 11 \\ 8 \end{bmatrix}$$

$$F_c = F \cdot M$$

$$G_c = F \cdot n = 0$$